The static spacetime relative acceleration for the general free fall and its possible experimental test

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Abstract

Mishra has recently established, using a generic static metric, the relative local proper-time 3-acceleration of a test-particle in one-dimensional free fall relative to a static reference frame in any static spacetime. In this paper, on the grounds of gravitoelectromagnetism we establish, in a covariant spacetime form, the relative 4-acceleration for the general free fall, indicating its canonical representation with its 3-space cinematical content. Then we obtain the relation between this representation and the very known expression for the relative free fall acceleration in Fermi coordinates. Taking this into account, it is shown that an experiment with relativistic beams in a circular accelerator, modelled by Fermi coordinates, recently proposed by Moliner et al, can test the here established covariant result and, therefore, can also verify Mishra's formula. This possibility of experimental verification, besides its intrinsic importance, can answer a recent inquire by Vigier, related to his recent proposal of derivation of inertial forces.

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1 Introduction

Mishra[1] has recently presented some relations (the "transformation law" and the "addition law" for accelerations) between cinematical observations on particles in one-dimensional motion in a general relativistic static spacetime, as performed by accelerated observers in this spacetime. From his general relativistic addition law Mishra obtains, in particular, the formula $\vec{a} = \gamma^{-2}\vec{g}$ (see equation (11) in [1]) for the local proper-time relative 3-acceleration $\vec{a}$ (Mishra's "physical acceleration") of a relativistic particle in one-dimensional free fall in any static gravitational field, relative to a preferred non-linear

static reference frame ($\vec{g}$ stands for the acceleration when the "physical" relative velocity $\vec{v}$ is zero; $\gamma^{-2} = (1 - v^2)$ in units in which $c = 1$).

More recently J.P.Vigier has pointed out the convenience of performing a laboratory test to verify this formula. His suggestion comes up in the course of an extensive article[2] dedicated to expose a possible, non-machian, solution to the old-age "unsolved mistery in modern physics" of the origin and nature of inertia. In his proposal, inertial forces arise from the local interaction of a physical vacuum or ether (in Dirac's covariant model) with accelerated particle-like solitons piloted by surrounding wave-packets. For simplicity Vigier considers only the one-domensional accelerated motion of the solitons in the ether and puts to work some inertial and accelerated observers/frames. Then the previous Mishra's formula, says Vigier, ..."plays a crucial role in our derivation/interpretation of inertia. It evidently proves that the acceleration due to a pseudo-force (inertial force) and that due to the force of gravity are both decreased by a factor $(1 - v^2/c^2)$..."

In this paper, we situate the problem in an enlarged context, following the gravitoelectromagnetism (GEM) formalism as outlined by Bini et al[3]. These authors have, in particular, re-obtained, in a coordinate-free form, all the results of Mishra.

Our first step is to extend, in a covariant spacetime form, Mishra's formula to the case of the static general free fall. Then we represent the obtained 4-covariant formula in a canonical form in the observers computational 3-space and proper-time. Further on, introducing observer-adapted Fermi coordinates (FC) we obtain the relations between the canonical representation and the well known general free-fall relative Fermi-coordinates acceleration $\vec{a}_{FC}$.

As we will see, $\vec{a}_{FC}$ does not coincide with the relative acceleration in the canonical representation and does not contain Mishra's one-dimensional expression. At contrary, the measurable physics which stems from $\vec{a}_{FC}$ for the one-dimensional free fall is "strange" and radically different from that one which comes from Mishra's law.

Indeed, for $\hat{v}_{FC} = \hat{g}$ it will be $\vec{a}_{FC} = d\vec{v}_{FC}/dx^0_{FC} = k\vec{g}$, with $k = (1 - 2v^2_{FC})$. Thus, only for $v_{FC} < 2^{-1/2}$ it will be $k > 0$. For $v_{FC} = 2^{-1/2}$, the particle will follow a FC-uniform movement and, when $v_{FC} > 2^{-1/2}$, it will be $k < 0$. Clearly, this kind of "anomalies" is absent from Mishra's expression[1].

[1] Jaffe and Shapiro[4] have already obtained the same "anomalies" for the spherically symmetric gravitational field in Schwarzchild coordinates.

The obtainment of a connection between the canonical relative 3-acceleration representation of the GEM spacetime formula and Fermi's relative 3-acceleration allows us to reconsider an experiment recently proposed by Moliner et al[5] with almost horizontal relativistic particles in a circular accelerator in order to test $\vec{a}_{FC}$, with Fermi coordinates modelling the experiment. In view of the previously obtained relations between the GEM formulae and $\vec{a}_{FC}$, we show that the realm of the experiment can be considerably enlarged to test also the GEM expressions and its theoretical implications, in particular Mishra's formula. This, besides to be a possible test of general relativity in a terrestrial experiment involving relativistic massive particles, would also be significant to estimate Vigier's derivation of inertial forces.

2 The GEM formula for the general free fall

2.1. Let us initiate with a brief recall on Bini et al exposition.

Spatial gravitational forces modelled after the electromagnetic 4-force, that is, "gravitoelectromagnetic forces", rely on the splitting of spacetime by means of a congruence of test-observers (u). The decomposition of each tangent space into a local direction along the 4-velocity vector field u of (u) and its orthogonal complement, the local instantaneous u-rest space LRS_u, induces a corresponding coordinate-free decomposition of all spacetime tensors and tensor equations, leading to spatial spacetime tensor fields (any contraction with u gives zero) and spatial equations which represent them, i.e., which "measure" them. This decomposition is accomplished by $T(u)$ and $P(u)$, the operators of temporal projection and of spatial projection into LRS_u, respectively, being $P(u)X = X + u[u \cdot X]$. These operators may be identified with suitable mixed second rank tensors acting by contraction.

Through $P(u)$ there are also introduced spatially projected differential operators: so, from the spacetime covariant derivation operator ${}^4\nabla$, it arises the spatial covariant derivative $\nabla(u) = P(u)\,{}^4\nabla$, the spatial Fermi-Walker derivative $\nabla_{fw}(u) = P(u)\,{}^4\nabla_u$ (which, for spatial tensor fields coincides with the spacetime Fermi-Walker derivative along u) etc.

Considering now a test-particle (U) with 4-velocity U, the orthogonal decomposition of U w.r.t u defines the relative velocity $v(U, u)$ of (U) w.r.t to (u) and the associated gamma factor, i.e.,

$$U = \gamma(U,u)[u + v(U,u)] \tag{1}$$

where

$$\gamma(U,u) = [1 - v(U,u) \cdot v(U,u)]^{-\frac{1}{2}} \tag{2}$$

(standing the dot for the inner product of spacetime vectors). $v(U,u)$ is u-spatial, since it is the rescaled u-spatial projection of U, i.e.,

$$v(U,u) = \gamma(U,u)^{-1} P(u) U \tag{3}$$

Now let $a(U) = \nabla_U U$ be the 4-acceleration of (U) and let $a(U) = \tilde{f}(U)$ be the equation of motion for U, being $\tilde{f}(U)$ the 4-force per unit mass on the test-particle U. Then the ortogonal decomposition and the spatial projection of the spacetime tensors and of the equation of motion lead to

$$A(U,u) = \tilde{F}(U,u) \tag{4}$$

where

$$A(U,u) = \gamma(U,u)^{-1} P(u) a(U) \tag{5}$$

(and equivalently for $\tilde{F}(U,u)$ and $\tilde{f}(U)$).

Expressing $A(U,u)$ in terms of the relative momentum per unit mass, $\tilde{p}(U,u) = \gamma(U,u)v(U,u)$, introducing the composite projection map defined by $P(u,U,u) = P(u)P(U)P(u)$, which is an automorphism of LRS_u, and defining

$$a_{fw}(U,u) = \gamma(U,u)^{-1} P(u) \nabla_U v(U,u) \tag{6}$$

as the Fermi-Walker relative acceleration of U w.r.t. u, Bini at al have derived the expression

$$A(U,u) = -\gamma(U,u)[g(u) + H_{fw}(u)v(U,u)] + \gamma(U,u)P(u,U,u)a_{fw}(U,u) \tag{7}$$

Here

$$g(u) = -a(u) \tag{8}$$

$$H_{fw}(u) = -\nabla(u)u = \omega(u) - \theta(u) \tag{9}$$

being $\omega(u)$ and $\theta(u)$, respectively, the vorticity (rotation) and expansion tensors of the observers family (u), and being implied a contraction between $H_{fw}(u)$ and $v(U,u)$.

Now, $a_{fw}(U,u)$ can be rewritten as

$$a_{fw}(U,u) = \frac{D_{fw}(U,u)}{d\tau_{U,u}} v(U,u) \tag{10}$$

where $D_{fw}(U,u)/d\tau_{U,u}$ is the Fermi-Walker total spatial covariant derivative along the world-line U expressed in terms of a parametrization corresponding to the sequence of differential proper-times of the observers (u) along the world-line U, so that

$$\frac{d\tau_{U,u}}{d\tau_U} = \gamma(U,u) \tag{11}$$

where $d\tau_U$ corresponds to the U-proper-time parametrization.

Besides, it can also be obtained

$$\frac{D_{fw}(U,u)}{d\tau_{U,u}} \tilde{p}(U,u) = \tilde{F}^G_{fw}(U,u) + \tilde{F}(U,u) \tag{12}$$

being

$$\tilde{F}^G_{fw}(U,u) = \gamma(U,u)[g(u) + H_{fw}(u)v(U,u)] \tag{13}$$

This allows us to interpret $\tilde{F}^G_{fw}(U,u)$ as the relative spatial gravitational force on U w.r.t. u, being $g(u)$ the gravitoelectric vector-force field and $H_{fw}(U,u)$ the gravitomagnetic one. (For a Minkowky spacetime in which (u) is an inertial observer, $\tilde{F}^G_{fw}(U,u)$ is zero.)

2.2. We will now extend Misrha's result, obtaining the expression for the relative 4-acceleration in the case of the general free fall.

Let us begin observing that, in our case, the test-particle U is free, so the 4-acceleration $a(U)$ is null. Thus we have $A(U,u) = 0$.

Besides, the spacetime is static and (u) is the prefered local reference frame, that is, u is the direction of a time-like Killing vector field, which implies that $\theta(u) = \omega(u) = 0$. So, $H_{fw}(u) = 0$ and (7) reduces to

$$P(u)P(U)P(u)a_{fw}(U,u) = g(u) \tag{14}$$

Taking now into account that, by definition, $a_{fw}(U,u)$ and $v(U,u)$ are u-spatial and using the orthogonal decomposition (1) of U, the succesive projections (14) lead to

$$a_{fw}(U,u) + \gamma^2[a_{fw}(U,u) \cdot v(U,u)]v(U,u) = g(u) \tag{15}$$

Now, multiplying this equation by $v(U,u)$ and considering the definition (2) of $\gamma(U,u)$, it is obtained

$$\gamma^2[a_{fw}(U,u) \cdot v(U,u)]v(U,u) = g(u) \cdot v(U,u) \tag{16}$$

or, finally,

$$a_{fw}(U,u) = g(u) - [g(u) \cdot v(U,u)]v(U,u) \tag{17}$$

which is the 4-covariant GEM expression for the general free fall in a static spacetime[2].

It is immediate to verify that, for $\hat{\nu}(U,u) = \hat{g}(u)$, it results

$$a_{fw}(U,u) = \gamma^{-2}g(u), \tag{18}$$

which is the spacetime covariant expression of Mishra's result for the one-dimensional free fall.

3 The canonical representation

Equation (17) as well as (18) previously derived are general covariant equations for the free-fall 4-acceleration relative to a preferred static reference frame where the observers constitute a time-like Killing vector field, always present in some open submanifold of any stationary spacetime. Since our generic spacetime metric is not only stationary but, more than that, static,

[2]This result can also be obtained directly by using the expression of $[P(u)P(U)P(u)]$ as given in line 1 of Table 1 of reference [3]. Or, alternatively, by solving (14) for $a_{fw}(U,u)$, using the expression of $[P(u)P(U)P(u)]^{-1}$ as given in line 2 of the table, as suggested by an anonimous referee.

the observers local rest space constitutes a spatial-like slicing orthogonal to the observers spacetime threading.

Thus we can complete the spacetime threading to an observer-adapted frame, that is, any frame $\{e_\alpha\}$, $\alpha = 0, 1, 2, 3$, such that e_0 is along the observer 4-velocity u and the spatial frame $\{e_i\}$, $i = 1, 2, 3$, spans the local rest space at each point along u. Besides, the spatial frame is Fermi-Walker transported along u, which assures that it remains orthogonal (and even orthonormal if we choose so). Finally we take for our present purposes the frame basis to be a coordinate-one, being $\{x^\alpha\}$ the local coordinates adapted to the frame.

Under these conditions, the general theory shows that the algebra of stationary spatial tensors is isomorphic to the tensor algebra of the computational 3-space equipped with the time-independent projected spatial metric γ_{ij} expressable in the local adapted coordinates[3,6].

Furthermore, since the $g_{\alpha\beta}$ metric is static, any spatial-like orthogonal slice can be taken as the computational space and $\gamma_{ij} = g_{ij}$. Besides, the spatial operators of static spatial fields reduce to the correspondent operators defined with respect to γ_{ij}. From hereafter a non-linear static reference frame will be always considered equipped with the above defined structures.

Now, returning to the 4-covariant GEM equation referred to such a static reference frame, $g(u) = -\nabla_u u$ and, since u is a unit vector, $g(u)$ is spatial and, by definition, $a_{fw}(U, u)$ and $v(U, u)$ also are. Then the projected equation can be noted in the 3-space vector notation

$$\vec{\tilde{a}} = \vec{g} - \vec{g} \cdot \vec{\tilde{v}}\vec{\tilde{v}} \tag{19}$$

where $a_{fw}(U, u) = (0, \vec{\tilde{a}})$, $g(u) = (0, \vec{g})$, $v(U, u) = (0, \vec{\tilde{v}})$ and the spatial inner product can be considered as arising from

$$X \cdot_u Y = P(u)_{\alpha\beta} X^\alpha Y^\beta = g_{ij} X^i Y^j = \gamma_{ij} x^i y^j \tag{20}$$

for any pair of spatial vector fields $X = (0, \vec{x})$ and $Y = (0, \vec{y})$. The projected equation (19), for its naturalness, will be said the *canonical representation* of the covariant GEM expression (17).

The generic relation between the projected variables $\vec{\tilde{v}}$ and $\vec{\tilde{a}}$ in the canonical representation and the relative 3-geometric cinematical variables is also given by the general theory[3,6], but it will be worthwhile for our immediate purposes to unfold it here directly.

Then let x_U^α be the coordinates of some particle world-line with 4-velocity U. So $\mathcal{U}^\alpha = dx_U^\alpha/dx^0 = \dot{x}_U^\alpha$ will denote the coordinate-components of the coordinate-velocity of U world-line, whose components are

$$U^\alpha = \frac{dx_U^\alpha}{d\tau_U} = \dot{x}_U^\alpha \left(\frac{dx_U^0}{d\tau_U}\right) = \Gamma_{x^\alpha}(U,u)\dot{x}_U^\alpha \tag{21}$$

being

$$\frac{dx_U^0}{d\tau_U} = (g_{\alpha\beta}\dot{x}_U^\alpha \dot{x}_U^\beta)^{-\frac{1}{2}} = (\mathcal{U}_\alpha \mathcal{U}^\alpha)^{-\frac{1}{2}} \equiv \Gamma_{x^\alpha}(U,u) \tag{22}$$

the x^α-coordinate Lorentz factor.

Let us now denote v^i the coordinates of a 3-vector $\vec{v}$ defined by

$$v^i = \frac{dx_U^i}{d\tau_{U,u}} = \dot{x}_U^i g_{00}^{-1/2} \tag{23}$$

since $dx_U^0/d\tau_{U,u} = g_{00}^{-1/2}$ in the static metric expressed in x^α-coordinates.

Note that this definition implies that

$$v^2 = \left(\frac{dl}{d\tau_{U,u}}\right)^2 \tag{24}$$

where $dl^2 = g_{ij}\, dx_U^i\, dx_U^j$. So $\vec{v}$ is the 3-velocity of the U-particle measured by the u-observer at rest in the same place as the U-particle, in the proper-time of the observer, to be called hereafter the local proper-velocity of the particle U.

Then, denoting $\gamma^*(U,u) = [1-v^2]^{-1/2}$, we have

$$\Gamma_{x^\alpha}(U,u) = g_{00}^{-1/2}[1 - g_{ij}v^i v^j]^{-\frac{1}{2}} = g_{00}^{-1/2}\gamma^*(U,u) \tag{25}$$

Thus $\gamma^*(U,u) = \gamma(U,u) = d\tau_{U,u}/d\tau_U$, as defined in (11), and

$$U^i = \gamma(U,u)v^i \tag{26}$$

$$U^0 = \gamma(U,u)g_{00}^{-1/2} \tag{27}$$

Therefore

$$v^i = \gamma^{-1}(U,u)U^i = \gamma^{-1}(U,u)[P(u)U]^i \qquad (28)$$

which, by definition of $v(U,u)$ (see (3)), leads to $v^i = v^i(U,u) = (0, \vec{\tilde{v}})^i$, thus identifing $\vec{\tilde{v}}$ with $\vec{v}$, that is, with the local 3-proper-velocity of the particle U relative to the stationary observer u.

Let us consider now the cinematical meaning of the spatial projection $\vec{\tilde{a}}$ of $a_{fw}(U,u)$ in the spatial geometry associated to the static reference frame. From definition (10) it comes

$$a_{fw}(U,u) = P(u)\frac{D}{d\tau_{U,u}}[\gamma^{-1}(U,u)P(u)U] = P(u)\frac{D}{d\tau_{U,u}}(0, \vec{\tilde{v}}) = (0, \vec{\tilde{a}}) \qquad (29)$$

so that we can identify the projected acceleration $\vec{\tilde{a}}$ with

$$(\vec{a})^i = \left(\frac{{}^3D\vec{v}}{d\tau_{U,u}}\right)^i = \left(\frac{d\vec{v}}{d\tau_{U,u}}\right)^i + \Gamma^i_{jk}v^j v^k \qquad (30)$$

where Γ^i_{jk} are the components of the Riemannian connection associated to the 3-metric g_{jk}. That is, we can identify $\vec{\tilde{a}}$ with $\vec{a}$, the local 3-proper-acceleration of the particle U relative to the stationary observer u.

So, we have shown that the canonical representation of the covariant GEM formula can be identified with the expression

$$\vec{a} = \vec{g} - \vec{g}\cdot\vec{v}\,\vec{v} \qquad (31)$$

where $\vec{a}$ and $\vec{v}$ have now a precise 3-geometric cinematical content.

Let us observe that an expression like (31) has been recently presented by Mould[7] for uniformly accelerated frames in the flat spacetime, starting from the specific metric to obtain the coordinate-acceleration and changing then to the suitable local proper-observers (see equation (8.45) in [7]). So, the present GEM derivation can be said to extend Mould's expression to any static spacetime and Mishra's result to the general free fall.

4 The GEM formulae and the cinematical variables in Fermi coordinates

Let us consider Fermi-coordinates adapted to a stationary observer modelled by a world-line u with covariant 4-acceleration $a(u) = -g(u)$. As these coordinates are suitable to model terrestrial experiments like the one considered in the next section, let us now try to connect the GEM formulae, through the canonical representation previously derived, with the cinematical variables in these particular coordinates.

For this purpose we need to establish the suitable relations between the local cinematical variables, in the canonical representation, and the adapted Fermi-coordinates cinematical variables. (Clearly, in this context, a preliminary assumption must be the Fermi-Walker transport of the Fermi spatial frame, in order to assure orthonormality and "non-rotation" of the frame.)

Locally, in Fermi coordinates, the spacetime metric will be, as it is well known (see equation (13.71) in [8])

$$ds^2 = (1 - 2g_i x^i_{FC})(dx^0_{FC})^2 - \delta_{ij} dx^i_{FC} dx^j_{FC} + O\left(|x^i_{FC}|^2\right) dx^\alpha_{FC} dx^\beta_{FC} \quad (32)$$

being $\alpha, \beta = 0,1,2,3$; $i, j = 1,2,3$; $g(u) = (0, \vec{g})$; $g_i = (\vec{g})_i$. Stationarity implies that $\vec{g}$ does not depend on x^0_{FC}.

This means that our observer has been immersed in a family of local stationary observers (u), whose local rest spaces integrate to a spatial-like hypersurface which is locally flat at this order of approximation, but whose coordinate clocks are all paced by our observer at the origin. Let us recall that, in the previous GEM equations, all the local observers at rest in the frame reparametrize any geodesic world-line U of a particle in free fall by their own local proper-time $\tau_{U,u}$ (not by the coordinate-time x^0 nor by the proper-time τ_U of U).

Now, let us apply (23) and (30) to the adapted Fermi coordinates. With the notation $v^i_{FC} = \dot{x}^i$, $a^i_{FC} = \ddot{x}^i$, one has

$$\vec{v} = g_{00}^{-1/2} \dot{\vec{x}} = (1 - 2g_i x^i_{FC})^{-\frac{1}{2}} \vec{v}_{FC} \quad (33)$$

$$\vec{a} = \frac{{}^3D\vec{v}}{d\tau_{U,u}} = \frac{d\vec{v}}{d\tau_{U,u}} \quad (34)$$

since the spatial 3-metric $g_{ij} = \delta_{ij}$ is flat. Then, from $d\tau_{U,u} = g_{00}^{1/2} dx^0_{FC}$ it will be

$$\vec{a} = g_{00}^{-1/2} \frac{d}{dx^0_{FC}} (g_{00}^{-1/2} \vec{v}_{FC}) = (1 - 2g_i x^i_{FC})^{-1} \vec{a}_{FC} + (1 - 2g_i x^i_{FC})^{-2} g_j v^j_{FC} \vec{v}_{FC} \tag{35}$$

For the observer at the origin we have

$$\vec{v} = \vec{v}_{FC} \tag{36}$$

$$\vec{a} = \vec{a}_{FC} + \vec{g} \cdot \vec{v}_{FC} \; \vec{v}_{FC} \tag{37}$$

which establishes the relations between the canonical cinematical variables and the Fermi-coordinate ones.

On the other hand, $\vec{a}$ is expressed by (31), so that, in view of (36),

$$\vec{a} = \vec{g} - \vec{g} \cdot \vec{v}_{FC} \; \vec{v}_{FC} \tag{38}$$

The consistence of the pair of equations (37) and (38) for $\vec{a}$ can be verified by obtaining from them the very well known expression

$$\vec{a}_{FC} = \vec{g} - 2\vec{g} \cdot \vec{v}_{FC} \; \vec{v}_{FC} \tag{39}$$

for the Fermi coordinate-acceleration, which is usually directly obtained by a very distinct derivation (see equation (13.75) of [8]).

Note the essential theoretical difference between $\vec{a}$ and $\vec{a}_{FC}$: $\vec{a}$ is constructed in a coordinate-free manner and so is invariant; the change of the temporal parametrization (from the observers proper-time to the Fermi coordinate-time) of the test-particle world line when we go from the canonical representation to the Fermi coordinates produces the variation $\vec{a} - \vec{a}_{FC} = \vec{g} \cdot \vec{v}_{FC} \; \vec{v}_{FC} \neq 0$, when $\vec{g} \cdot \vec{v}_{FC} \neq 0$ (or $g_i x^i_{FC} \neq 0$), since clocks at different heights beat at different rates (otherwise, $\vec{a} = \vec{a}_{FC} = \vec{g}$). (In what concerns spatial coordinates, let us recall that the Fermi ones measure proper-distances, in this order of approximation.)

Clearly, from the expression for $\vec{a}_{FC}$, changing from coordinate-time to local proper-time, it would be possible to derive $\vec{a}$, but this would hidden the canonical content of $\vec{a}$ and of equation (31) as the canonical 3-space

representations of the spacetime covariant Fermi-Walker relative acceleration $a_{fw}(U,u)$ and of the covariant equation (17), respectively - that is, the covariant content of the problem.

5 A possible test for the GEM expressions

Moliner et al[5] have recently suggested a possible way to test the Fermi coordinate-acceleration expression. In their suggestion, such a test is to be performed in a circular accelerator where a charged particle moves under the influence of suitable electric and magnetic fields, besides, of course, the Earth gravitational one.

Then, equation (39) is extended to give

$$\vec{a}_{FC} = \vec{g} - 2\vec{g}\cdot\vec{v}_{FC}\vec{v}_{FC} + \frac{e}{m\Gamma_{FC}(U,u)}(\vec{E} + \vec{v}_{FC}\times\vec{H} - \vec{E}\cdot\vec{v}_{FC}\vec{v}_{FC}) \quad (40)$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, e is the charge, m is the mass and $\Gamma_{FC}(U,u) = (1-v_{FC}^2)^{-1/2}$ is the Fermi-coordinate Lorentz factor.

Defining

$$\vec{E}_p = \vec{E} + \frac{m\Gamma_{FC}(U,u)}{e}\vec{g} \quad (41)$$

equation (40) can be rewritten in the form

$$\vec{a}_{FC} = \frac{e}{m\Gamma_{FC}(U,u)}(\vec{E}_p + \vec{v}_{FC}\times\vec{H} - \vec{E}_p\cdot\vec{v}_{FC}\vec{v}_{FC}) - \vec{g}\cdot\vec{v}_{FC}\vec{v}_{FC} \quad (42)$$

being $-(m\Gamma_{FC}(U,u)/e)\vec{g}$ the electric field necessary to prevent the particle falling down.

Taking for $\vec{H}$ a uniform magnetic field and for $\vec{E}_p$ a periodic electric field, both in $\hat{g}$-direction, the term $-\vec{g}\cdot\vec{v}_{FC}\,\vec{v}_{FC}$ leads to a measurable horizontal drift of the trajectory, in consequence of a ressonance effect arising from making the frequency of $\vec{E}_p$ equal to the Larmor frequency of the particle in the magnetic field.

If $\vec{E}_p = 0$, the particle movement will be strictly horizontal and circular, since then $\vec{E}$ would only prevent the following down. Switching the additional

electric field $\vec{E}_p$, a vertical periodic component is summed up to the movement and the ressonance effect can arise. The final result is that the trajectory of the particle projected in the horizontal plane is now a drifting circle. The final vertical velocity comes from $\vec{E}_p$ and from the gravitational acceleration arising from the term $\vec{g} \cdot \vec{v}_{FC}\ \vec{v}_{FC}$. This velocity also contributes to the horizontal component of the particle acceleration through the horizontal part of this same term.

This way it seems possible to verify experimentaly the Fermi coordinate-acceleration. Remembering that $\vec{a}_{FC}$ can be correlated, through the canonical representation, to the GEM formula for the general free fall and that the latter contains Mishra's result, we are led to reconsider the Moliner et al experiment as a possible way to test also both these expressions.

Surely, the following question can be posed: using a circular accelerator in Moliner et al experiment, why is that we are obliged to work (and measure) $\vec{a}_{FC}$ instead of working and measuring directly $\vec{a}$? The situation is as follows:

a) in the canonical representation, the trajectory of the particle is time-parametrised by the proper-time clock of the observer at rest at the spatial position of the particle. At each position, $\vec{a}$ is local (locally measured), since it refers to an arbitrary small spatial neighbourhood during an arbitrary small interval of time. From the local point of view, all the observers are mutually independent, none is preferred and each one measures $\vec{a}$ at its position (in an arbitrary small neighbourhood). But one cannot characterize a circular trajectory in such a neighbourhood;

b) to characterize (to measure) a circular trajectory in the accelerator one must refer to fixed, finite, spatial parameters (e.g., the radius, cartesian coordinates etc), which cannot be captured (measured) in such arbitrary small neighbourhoods, so one will necessarily be led to define non-local simultaneity (i.e., Einstein's sincronization plus a common rate of the finitelly separated clocks);

c) if the accelerator experiment deals only with strictly horizontal circular trajectories, then all the proper-time clocks at rest at the same height are equivalent (they beat at the same rate), so the finitelly separated clocks are already naturally coordinated and there is no problem: since then $g_i x^i_{FC} = 0$, i.e., $\vec{g} \cdot \vec{v}_{FC} = 0$, one will have (measure) strictly

$\vec{a} = \vec{a}_{FC} = \vec{g}$ and no drift. But this implies only the first half of Moliner et al experiment;

d) to reach the entire scope of the experiment, the trajectory of the test-particle, by necessity, must be strictly non-horizontal: in fact, it is made to oscilate vertically w.r.t. the horizontal circunference of reference, so one has not anymore a natural time coordinisation between the clocks at different heights. But, as we have seen in b), such a coordinisation is unavoidable for a (global) characterization of the trajectory (besides, the drift is also a non-local effect to be measured). So the observers at different heights made the gentlemen agreement to pace their clocks by the clock at the reference level and, with this agreement, what their devise will measure will be $\vec{a}_{FC}$ (which differs from $\vec{a}$ by $\vec{g} \cdot \vec{v}_{FC}\vec{v}_{FC}$) and a non-null drift of the circular horizontal projection of the particle trajectory.

Note that if one tries to reconstruct the theoretical scheme of Moliner et al experiment (that is, equations (40)-(42) etc) by using directly $\vec{a}$ instead of $\vec{a}_{FC}$, then the term $\vec{g} \cdot \vec{v}_{FC}\, \vec{v}_{FC}$ automatically desappears from (42) and the theoretical scheme becomes vacuous. So the scheme is consistent just with the empirical, observable, content referred above in a) and b).

In face of this fact that Moliner et al experiment does not test directly the GEM expression or Mishra's result, we must reexamine to what extent the physical content effectively involved in the test really includes them.

The following reasoning can be done:

a) in the general GEM formula, and in its canonical representation, a_{fw} (resp. $\vec{a}$) is a sum of two parcels, one according to $\hat{g}$ and the other according to $\hat{v}$. Its reduction to Mishra's expression comes for $\hat{v} = \hat{g}$, ie, is made up of the contributions of both these parcels. So, any experiment able to verify separately each parcel in a_{fw} (resp. $\vec{a}$) confirms the entire general formula and, in particular, Mishra's expression;

b) the comparison between $\vec{a}$ and $\vec{a}_{FC}$ ((31) and (39)) shows that each parcel of a_{fw} (resp. $\vec{a}$) corresponds to the respective parcel of $\vec{a}_{FC}$. So, any experiment able to verify separately both parcels of $\vec{a}_{FC}$ implies the same kind of verification for a_{fw} (resp. $\vec{a}$) (and reciprocally) and so confirms the GEM expression and, consequently, Mishra's equation;

c) fortunately, the Moliner et al experiment tests, separately, each parcel of $\vec{a}_{FC}$: the measurement of the electric field necessary to avoid the charge falling down during the circular movement tests the parcel according to $\hat{g}$; and the measurement of the horizontal drift velocity tests the parcel according to $\hat{v}$.

Thus we can conclude that, if the experiment gives both the predicted electric field and the forseen drift velocity, it will confirm the generic static GEM expressions for the general free fall and, in particular, Mishra's one-dimensional result. Besides, it will favour the soundness of Vigier's solution on the nature of inertial forces.

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